In the study of fully developped turbulence, what we call the canonical formalism in this document (to make a precise distinction with the microcanonical formalism developed in GEOSTAT) has been established with solid foundations. From all aspects, and specially from a dynamical systems perspective, the microcanonical formalism studied in GEOSTAT is completely different by nature, most notably in the situation far from the statistical equilibrium, which is the most common case in all complex signals, and where the moments of significant variables are not easily obtained. The MMF can be seen as an extension of the canonical formalism, in the sense that the latter lies on purely statistical descriptions.

To dwelve into the realm of a description outside statistical equilibrium, a fundamental concept is given by the notion of singularity exponent. Singularity exponents can be computed in many different ways. In particular, it is a notion that is dependent of the formalism used (canonical or microcanonical). This distinction is of fundamental importance. In the so-called canonical formalism, the exponents are obtained through a family of expected values associated to operators $\mathbb{T}_r$ depending on the scale **r**:

$$\langle|\mathbb{T}_{\mathbf{r}}\mathbf{s}|^p\rangle = \alpha_p\,\mathbf{r}^{\tau_p} + o\left(\mathbf{r}^{\tau_p}\right)$$

(3)

and the coefficients p are then related to the Legendre spectrum of the canonical multiplicative cascade. In the **microcanonical formalism**, the singularity exponents are not computed through the values of moments, but depend instead on the spatial location of a point in the signal domain:

$$\mathbb{T}_{\mathbf{r}}\mathbf{s}(\mathbf{x}) = \alpha(\mathbf{x})\,\mathbf{r}^{h(\mathbf{x})} + o\left(\mathbf{r}^{h(\mathbf{x})}\right)\ (\mathbf{r}\to 0)$$

(4)

As soon as coordinates (spatial or temporal, or depending on the characteristics of the signal's domain) are introduced, geometric super-structures can be defined; they are naturally associated to the singularity exponents, the multiscale hierarchy, and can also be related to the singularity spectrum defined in the canonical formalism . A fundamental aspect in GEOSTAT is to study their relationships to the dynamics of the underlying complex system. One of these geometric entities, the so-called most singular manifold plays a central role in the framework of reconstructible systems, because of its signification as a set of transition fronts. The MMF, through the reconstruction formula, therefore provides a theoretical background for the study of complex signals of different types (turbulence, geophysical fluids, oceanographic, meteorologic and astronomic data, speech signal and biological datasets). Hence the scientific and methodological justification for the creation of an INRIA project on these themes.