Scientific Foundations

Fundamentally, GEOSTAT explores new methods for analyzing and understanding complex signals in different applicative domains through the theoretical advances of the **MMF (Microcanonical Multiscale Formalism)**, and the framework of **reconstructible systems**. The underlying theoretical results that motivates the use of the methodologies developped in GEOSTAT, and their application to different types of signals, is found in advances around renormalization methods in Physics, and the emergence of the notion of **universality class**. Derived from ideas in Statistical Physics, the methods developped in GEOSTAT offer new ways to relate and evaluate quantitatively the local irregularity in complex signals and systems, the statistical concepts of information content and most informative subset . That latter notion is developed through the notion of transition front and Most Singular Manifold .

As a result, GEOSTAT is aimed at providing radically new approaches to the study of signals acquired from different complex systems (their analysis, their classification, the study of their dynamical properties etc.). The common characteristic of these signals, as required by universality classes , being the existence of a multiscale organization of the systems. That point will be explained in this document. For instance, the classical notion of edge or border , which is of multiscale nature, and whose importance is well known in Computer Vision and Image Processing, receives, through the MMF, profound and rigorous new definitions. Used in conjunction with appropriate reconstruction formula , the MMF is capable of generalizing in a consistent manner the notion of edge so that the generalized definition is adequate to the case of chaotic data. The description is analogous to the modelling of states far from equilibrium, that is to say, there is no stationarity assumption. From this formalism we derive methods able to determine geometrically the most informative part in a signal, which also defines its global properties. In this way, the MMF allows the reconstruction, at any prescribed quality threshold, of a signal from its most informative subset, and is able to quantitatively evaluate key features in complex signals (unavailable with classical methods in Image or Signal Processing). We are then able to define, in an unprecedented rigorous and precise manner, the notion of transition front in a signal, which is much more complex than previously expected and, most importantly, related to multiscale notions encountered in the study of non-linearity. For instance, we give new insights to the computation of dynamical properties in complex signals, in particular in signals for which the classical tools for analyzing dynamics give poor results (such as, for example, correlation methods or optical flow for determining motion in turbulent datasets).

Objectives

Given that brief account, the problematics in GEOSTAT can be summarized at first glance in the following items:

- the accurate determination in any n-dimensional complex signal of singularity exponents at every point in the signal domain (as opposed to global and non-localized exponents in other classical approaches). This accurate determination is an extremely complex problem, related to properties associated to non-linearity and non-equilibrium states in analogies coming from Statistical Physics [3] . The singularity exponents give information about local power-law transitions around a point, they are related to a generalized notion of information content and transition front , and are defined from analogies observed in the behaviour of intensive variables around critical points in complex systems .
- The geometrical determination and organization of singular manifolds associated to various transition fronts in complex signals, the study of their geometrical arrangement, and the relation of that arrangement with statistical properties or other global quantities associated to the signal.
- The study of the relationships between the dynamics in the signal and the distributions of singularity exponents.
- The study of the relationships between the distributions of singularity exponents and other quantities associated to predictibility in complex signals and systems, such as cascading properties, large deviations and Lyapunov exponents.
- The ability to compute optimal wavelets and relate such wavelets to the geometric arrangement of singular manifolds and cascading properties.
- The translation of recognition , analysis and classification problems in complex signals to simpler and more accurate determinations involving new operators acting on singular manifolds using the framework of reconstructible systems.

In the applicative domain, GEOSTAT will focus its research activities to the study of three main classes of signals: remote sensing satellite acquisitions in Oceanography (study of different phenomena -i.e. geostrophic or non-geostrophic- complex oceanic dynamics, mixing phenomena, ocean/climate interaction), Speech processing (analysis, recognition, classification), signals in Astronomy (multi-dimensional implementation of the MMF, analysis of solar data, atmospheric perturbation of acquisitions with optical devices, interstellar medium).