

The main strength of the MMF consists in its ability to compute accurately the value of a singularity exponent around any point  $x$  in the domain of a complex signal  $s$ . The following, which is one of the functions used in implementing the MMF (precisely, in this case, a measure in the signal domain) is defined through the density of a generalized gradient:

$$T_{\mu}(x) = \int_{B_{\mu}(x)} \|\nabla s\|(\mathbf{y}) d\mathbf{y} \quad (5)$$

and whose *singularity exponents*  $h(x)$  derived from the behaviour described in equation (4) can be computed by appropriate wavelet projections (ball of radius  $r$  centered at point  $x$ ). A multiscale hierarchy directly related to *information content* (and, in the case of turbulence, to cascading properties) is defined from the distribution of the *singularity exponents*. That distribution being bounded from below for physical signals, a specific geometric super-structure, the Most Singular Manifold is the geometrical set associated to the lowest value :

$$\mathcal{F}_{\infty} = \{x, h(x) = h_{\infty}\}. \quad (6)$$

In the framework of reconstructible systems the set  $\mathcal{F}_{\infty}$  is shown to correspond to the statistically most informative part in the signal, and, consequently, an operator can be defined to recover the whole signal from its restriction to the Most Singular Manifold

$$s(x) = \hat{s}(s|_{\mathcal{F}_{\infty}})(x) \quad (7)$$

and the operator  $\hat{s}$  can be completely specified (usually in Fourier space) from physical considerations about processes. The framework of reconstructible systems opens the way to a whole area of research, for instance for the problem of motion analysis in oceanographic acquisition datasets. In contrast to conservation methods in Image Processing (optical flow, correlation methods etc.) the reconstruction formula permits the determination of the dynamics from one single acquisition in a temporal image sequence. See figure 1.

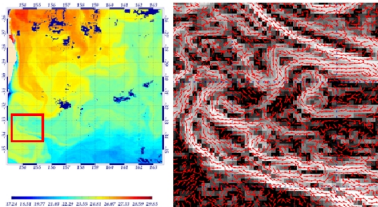


Figure 1. Left: excerpt from a Sea Surface Temperature (SST) image acquired by Modis. The image is in false colors and the value of a pixel records the temperature, in Celsius degrees, of the sea surface's upper layer. The image shows the coherent structures and turbulent aspects of the oceanic flow. In red is an excerpt, specifically chosen, containing important turbulent motion (the turbulent character can be evaluated, for instance, from the values of Lyapunov exponents). Right : application of the MMF, optimal wavelets and reconstruction formula lead to a proper determination of the motion field using only one image in the sequence. The background records the value of singularity exponents. The vector field is depicted in red in the foreground, renormalized to unitary vectors. The proper determination of turbulent motion in real acquisitions like this one shows one of the strong potential of the MMF.