In the past decades, the emergence of renormalization methods in Physics has led to new ideas and powerful multiscale methods to tackle problems where classical approaches failed. These concepts encompass a wide range of physical systems and they underline fundamental ideas:

- collective behavior of microscopic degrees of freedom, revealed by the study of fluctuations and statistical correlations ,
- critical phenomena observed at any scale, associated to critical divergence of mascroscopic variables described by scale laws and singularity exponents ,
- scale homogeneity and scale separation are replaced by the more general concept of scale invariance, and, as a consequence, the definition of hierarchical geometric structures that correlate the different scales in a system,
- the fundamental concept of universality class.

In the typical example of critical transitions, critical divergences are naturally associated to intensive variables, they indicate dramatical changes in the physical state of the system, and they are characterized by the values of singularity exponents. One observes divergence phenomena in correlation lengths and the presence of statistical fluctuations at all the scales. These phase transitions occur at a certain temperature, called the critical temperature θ_c and the description of a physical quantity X around the reduced temperature $\tau = \frac{\theta - \theta_c}{\theta_c}$ is governed by the value of the singularity exponent:

$$h = \lim_{\tau \longrightarrow 0} \frac{\log |X(\tau)|}{\log |\tau|}$$

(1)

hence

$$X(\theta) = \alpha(\theta - \theta_c)^h + o\left((\theta - \theta_c)^h\right)$$

(2)

Scale invariance is a natural consequence of this phenomenon and it implies that the functionnal dependency of variables remains unchanged under scale change. Fluctuations propagate through all scales in the same way, and microscopic dynamics become irrelevant. To properly characterize a critical point, it is necessary to determine as precisely as possible the value of the exponent, and one of the main difficulties is to determine the appropriate model to compute the exponents. The importance of that determination, which moreover makes one of the important scientific motivations in GEOSTAT, comes from the fact that an important number of relations between the exponents have their origin in thermodynamical considerations, they go beyond the consideration of a particular system . In this context, it becomes natural to search for such singularity behaviours in a wide range of acquired datasets. Recently, it has been understood that systems characterized by same singularity exponents form an universal class , as explained by renormalization techniques: in the vicinity of a critical point, the microscopic dynamics vanish to leave the place to macroscopic components precisely determined by the exponents.

singularity exponents are **universal**, they do not depend on the microscopic description of the system, but only of global properties such as its dimension and the range of the interaction. In Statistical Physics, universality appears in the following manner: very different systems have the same exponents, a notion that can be explained by renormalization : near a critical point fluctuations and disturbances occur at all scales, and thus a correct description is given by a scale invariant theory. This modern meaning of universality was introduced in L. Kadanoff et al. , and has found applications in other fields outside Physics, such as distributed systems and multi agents systems . In GEOSTAT the notion of universality class forms the cornestone that allows the determination of fine dynamical properties of signals and systems associated to scale invariant phenomena with the help of generalized singularity exponents . In other words, the singularity exponents (and the geometric notions they are associated with) give information and radically new methods for the analysis of signals associated to acquisition of different physical systems.

The determination of a singularity exponent at every point of a signal's domain is a subject of interest. The notion of reconstruction was verified in the case of certain meteorological datasets , in econometric time series and most importantly in oceanographic datasets , a domain which is an area of intense research between GEOSTAT partners. The study of ocean dynamics is of particular interest in the thematics developed in GEOSTAT, because a great amount of work has been devoted to establish a relationship between a super-geometric structure defined from the singularity exponents and a proxy stream function of the oceanic flow. These results are presently extented in many directions, notably for the determination of energy exchanges between the ocean and the atmosphere. Moreover, the potential of the geometric super-structures to recover dynamic variables from the data has been explored in the meteorological framework to determine rain precipitation in thermal infrared Météosat data. These last examples in Oceanography and Meteorology underline a key aspect in GEOSTAT: our goal is to link specific information in a complex signal to physical descriptions meaningful in terms of geometry and statistics: for example, domains related to GEOSTAT include multiplicative cascading, singularity analysis, transition front detection. Note also that GEOSTAT is not intended to be another project on multifractal analysis.

However the real world shows that the sole consideration of singularity exponents is not sufficient to assess the dynamical properties. One has to study the continuum of their organization. This is the entry point for the consideration of geometric structures: the realization of critical manifolds is a research theme under investigation in various contexts since many years, and the resolution of turbulent dynamics is not assessed by a single geometrical interface because, in a famous paper, Parisi and Frisch have established the relationships between the spectrum of singularity exponents associated to structure functions and the geometrical hierarchy predicted in Fully Developped Turbulence (FDT). Henceforth, the different geometrical super-structures organize themselves according to the dynamics to produce the singularity exponents observed in signals. From thereon, the interest for singularity exponents has grown firmly, and a great deal of research has been devoted to relate them to the statistical content, in various contexts.

In many different branches of applied sciences, sensors are delivering datasets that reproduce, at increasing spatial and temporal resolutions, the overwhelming complexity of natural phenomena. Here are some examples:

- the gigantic increase (in terms of spatial, temporal and spectral resolutions) of satellite data,
- the considerable increase of bandwith for the network delivery of high quality speech or image data,
- new imagery techniques in astronomy,
- new techniques in microscopy for biologic imagery,

These examples are not exhaustive, we mention them only to underline the new kind of research problems they raise to classical methodologies in statistical modeling and processing: the ever increasing complexity present in these datasets is not merely a quantitative hurdle that will be solved by considerations bound to mere computing power or hardware progress; they relentlessly lead to question the very pertinence of these classical methodologies, and they ask for new methodologies able to cope with the apparition of processes and phenomena which were not, up to now, accessible in the datasets . Most interestingly, these progresses in the quality of acquired datasets match the advances of new concepts from Statistical Physics, such as for instance in the generic power of renormalization methods for the characterization of multiscale phenomena around critical points in universality classes, and multiresolution analysis. This point of view forms the distinctive approach in GEOSTAT. For instance, new types of high resolution data in satellite imagery (in various spectral domains, even microwave for altimetry with the coming advent of SWOT data in 2012) challenge classical segmentation and matching methods; turbulence phenomena directly available in oceanographic datasets raise new methodologies for the assessment of the dynamics of the oceans. The same remarks apply in astronomy (e.g. the MUSE project) and speech processing.

GEOSTAT theoretical advances are focused, for one part, on the definition of reconstruction formula which allow the restitution of a signal (spatial or temporal predictibility) from knowledge about that signal on certain subsets known as geometric super-structures , of statistical importance (hence the name of the GEOSTAT proposal: Geometry and Statistics in acquisition data). These geometric super-structures are, of course, derived from singularity exponents : they convey the most significant information (in a statistical sense) across the scales in the system. This proves, at least empirically in an effective way, the signifiance of singularity exponents (as they are defined in a microcanonical way in the MMF). The information content associated to these geometric sets, their relationships with the class of reconstructible systems unveils the definition of new methods for recognition, classification and statistical modeling. This is a key aspect in GEOSTAT, which is justified by the physical signification of singularity exponents in universality classes: for instance, the reconstruction formula permits the characterization of fine phenomena in turbulent signals (see, for instance, figure 1). In GEOSTAT we are studying the introduction, in classification and recognition methods, of fine multiscale parameters derived from the MMF in complex signals such as speech or astronomical data. The theory of reproducing kernel Hilbert spaces, developped within the framework of reconstructible systems and the MMF, also has a strong potential in many domains such as approximation theory, recognition and classification. In **GEOSTAT**, reproducing kernels will be studied in conjunction with the MMF for recognition and classification, and serve as the basis for new reconstruction formula.

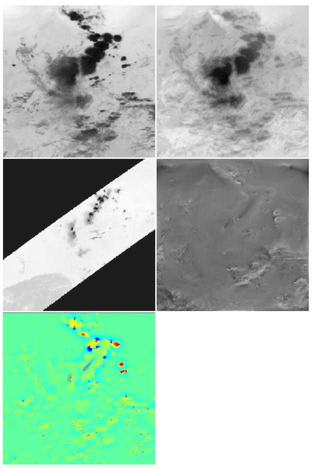

Figure 1. Top: excerpt from a MétéoSat infrared image acquired on 31-07-1998 at 16.00 GMT above West Africa (left) and reduced signal (right). Middle: Left: corresponding image acquired in the hyper-frequency range by the TRMM satellite, displaying ice crystals at the top of clouds, associated to precipitation. Ice crystals correspond to darkest pixels. Right: modulus of the multiscale source field, computed within the framework of reconstructible systems, shown in logarithmic scale. Zeros and poles are respectively associated to black and white pixels. Bottom: divergence of the velocity field estimated by classical optical flow. In blue: positive divergences, in red: negative divergences. There is a clear correspondance between divergence's extrema and zeros and poles of the source field. The source field is computed from one single acquisition, as opposed to optical flow. This example shows some of the MMF's potential.

In Image Processing and Computer Vision, a fundamental notion is found in the concept of edge or border, which is the building block of many other attributes, static or dynamic. A major advance in GEOSTAT is to redefine this fundamental notion to give it, through the MMF, a considerable extension well adapted to the case of multiscale data, and relate it to the notion of information content and transition front in a such a way that it finds a natural link with the notion of critical points in Statistical Physics. As a consequence, the notion of transition front in turbulent data can be quantitatively and precisely defined, and the evaluation of the dynamics in complex images associated to turbulent signals can be done with algorithms completely different, in nature, to those classically used in Image Processing, such as the optical flow or correlation methods and neural networks. These latter methods, which work quite well for rigid or elastically deformable objects, raise serious difficulties in the case of geophysical signals such as oceanographic images. These hurdles constitute an important topic of research and they are investigated, among other topics, for example by the FLUMINANCE project at INRIA, which has developped sophisticated methods to analyze motion in turbulent data -different in nature to those proposed in GEOSTAT-. In Oceanography, it is a very difficult problem to assess the motion of turbulent and coherent structures such as vortices or thermal fronts from satellite maps. A major advance in GEOSTAT is to use the MMF to obtain dynamical attributes using only one single snapshot in temporal sequences. Moreover the evaluation of dynamical properties using the MMF shows accurate numerical stability with satellite data, which are plagued by many artifacts related to the acquisition process.