Motion analysis in oceanographic satellite images using multiscale methods and the energy cascade

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Abstract

Motion analysis of complex signals is a particularly important and difficult topic, as classical Computer Vision and Image Processing methodologies, either based on some extended conservation hypothesis or regularity conditions, may show their inherent limitations. An important example of such signals are those coming from the remote sensing of the oceans. In those signals, the inherent complexities of the acquired phenomenon (a fluid in the regime of fully developed turbulence—FDT) are made even more fraught through the alterations coming from the acquisition process (sun glint, haze, missing data etc.).

The importance of understanding and computing vector fields associated to motion in the oceans or in the atmosphere (e.g.: cloud motion) raises some fundamental questions and the need for deriving motion analysis and understanding algorithms that match the physical characteristics of the acquired signals. Among these questions, one of the most fundamental is to understand what classical methodologies (e.g.: such as the various implementations of the optical flow) are missing, and how their drawbacks can be mitigated. In this paper, we show that the fundamental problem of motion evaluation in complex and turbulent acquisitions can be tackled using new multiscale characterizations of transition fronts. The use of appropriate paradigms coming from Statistical Physics can be combined with some specific Signal Processing evaluation of the microcanonical cascade associated to turbulence. This leads to radically new methods for computing motion fields in these signals. These methods are first assessed on the results of a 3D oceanic circulation model, and then applied on real data.

1. Introduction and state of the art

The analysis of motion information in temporal sequences of remotely sensed data constitutes an important theoretical and applicative domain in Computer Vision. In Oceanography (and Earth Observation) for example, the wealth and availability of temporal sequences of remotely sensed images at various spatial, temporal and spectral resolutions have deeply renewed the field [1]. These temporal image sequences convey key information about ocean dynamics (motion of passive tracers, vertical mixing etc.) [2–6]. Unfortunately, most existing methods in analyzing motion in Computer Vision do not take into account key features of ocean dynamics: turbulence and intermittence, which contribute importantly to the shape and motion of observed and acquired coherent structures. Besides, the specificities of these remotely sensed acquisitions (sun glint, haze, missing data etc.) lead to question the pertinence of existing Computer Vision approaches to analyze motion in these types of image sequences. Fig. 1 illustrates some of the difficulties inherently bound to these images.

Among the existing algorithms proposed for the motion analysis of fluid flow in temporal sequences, one finds methodologies based on correlation methods that try to estimate a vector field assumed to be locally constant in small windows $W(x)$ centered at various pixels $x$ in the image. The correlation is expressed by defining a similarity criterion, which can be implemented in Fourier space, leading to simple computational algorithms. These techniques are sometimes known under the name PIV (particle image velocity) for the visualization of motion flow [7]. We also mention particularly in our context the correlation-relaxation-labeling method [8] that was developed for SST motion. However, the locality assumption they use does not match the physical reality of turbulent fluid flows in remotely sensed images of the oceans, making them inappropriate in an operational context, and it is difficult to incorporate an a priori knowledge about the motion.

Another vast class of algorithms that has led to considerable extent these past few years, notably for the analysis of flow motion, hinges on luminance conservation hypothesis coupled with various types of regularity constraints. Basically, these
methods consist in minimizing an energy functional defined in the image domain $\Omega$ and made of two terms:

1. an energy functional expressing luminance conservation during motion, which leads, when written in differential form, to an apparent motion constraint,
2. a regularization term expressing spatial constraints on the set of possible solutions.

A typical example for the apparent motion constraint is the classical

$$E_1(l,v) = \int_{\Omega} \phi_1 \left( \nabla l(x) \cdot \nabla v(x) + \frac{\partial l}{\partial t}(x) \right) \, dx$$

($x$: pixel location, $\Omega$: domain in the image plane, $l$: grey-level intensity, $v(x)$: vector field (function of spatial position $x$), $\phi_1$: attenuation function (often the $L^2$ norm) [9,10]), while
regularizing terms are often written in the form of integrals of spatial derivatives of the vector field \( \mathbf{v}(\mathbf{x}) \). Generically, the regularizing term has the form

\[
E_r(\mathbf{v}) = \varepsilon \int \phi_2 \left( \mathbf{v}(\mathbf{x}), \ldots, \frac{\partial^2 \mathbf{v}(\mathbf{x})}{\partial x_i \partial x_j}, \ldots \right) \mathrm{d}x
\]

(\varepsilon \in [0, 1]; \text{weighting parameter, } \phi_2: \text{attenuation function}) It is not possible to review here the immense literature about optical flow, but we notice that, in relation with the case of turbulent and irregular motion we are interested in, and since the seminal work of Horn and Schunk [11], considerable work has been done to overcome some of the limitations inherent to the approach, notably in extending the formalism to the framework of multisresolution representation of data (to try to lessen the small displacement hypothesis) [12–16], and in modifying the regularizing energy functional to allow more irregular vector field solutions [17–20]. See also [60–65]. Notable extensions of this approach, intended to better approximate fluid flow motion, consisted in deriving new conservation laws starting from continuity equations in the motion of fluids [21]. Algorithms that exploit those ideas can be implemented in an operational context [22] but they need, however, precise pre-processing calibration. In some particular types, an exact conservation law can be derived for the apparent motion of the fluid [23]. Another important extension to luminance conservation models has also been geared towards the development of div-curl regularization, which basically consists in modifying the regularization term to incorporate higher order derivatives of \( \mathbf{v}(\mathbf{x}) \) related to divergence and rotational phenomena associated to fluid motion [24,25]. However, fundamentally, in the case of turbulent fluid flow motion and the complex phenomena associated to remote sensing acquisition processes (atmospheric correction, missing data, solar glint, haze etc.), any luminance conservation hypothesis can legitimately be strongly questioned, most notably if a large number of temporal acquisitions are to be used to evaluate the vector field: the variations in luminance from one frame to another are highly unpredictable, and regularizing conditions do not reflect physical properties specially in the case of large Reynolds numbers [26].

Parameter estimation methods have also been proposed for dense motion estimation, like those based on vorticity discretization [27,28]. They have been extended to generate fields that try to better satisfy the Navier–Stokes equations, by estimating a vector field satisfying an \textit{a priori} evolution law [29]. The determination of characteristic points or features used by these various methodologies often makes assumptions on regularity of the flow, like in [30]. Similarly, when contour evolution of deformable models is sought, like in [31], the level-set formulations used in the evolution model are only non-multiscale approximations of coherent structures characteristics and motions, written with level-set evolution models that do not take into account the turbulent nature of the flow. Paris and Frisch [32], in a very important paper, have put into evidence, in the case of FDT (fully developed turbulence, i.e. turbulent fluid motion with very high Reynolds Number [33]), a key observation of particular interest in the analysis of the motion in oceanographic image sequences acquired by satellites. Indeed they showed a fundamental relationship between a complex arrangement of hierarchical geometric structures associated to flow motion and the spectrum of singularity exponents related to structure functions. Consequently, it makes sense to obtain information about flow motion from the knowledge of the singularity spectrum. This has motivated the development of WTMM (Wavelet Transform Modulus Maxima) methodology by Arneodo and collaborators [34–36] which is fundamentally statistically based, (i.e. \textit{singularity exponents} are not computed at each pixel location \( \mathbf{x} \), but are related to global expectations of intensive physical variables).

To extend the analysis of turbulent flows towards an evaluation of singularity exponents at each point \( \mathbf{x} \) in the signal domain, it has been shown recently [37–39] that the notion of transition in acquisition signals of complex and turbulent phenomena is related to the notion of information content in a way that necessitates a finer microlocal analysis of the natural multiscale functionals associated to these signals, such as the multiscale measures

\[
\mu(E_r(\mathbf{x})) = \int_{E_r(\mathbf{x})} \| \nabla \mathbf{s}(\mathbf{y}) \| \mathrm{d}y
\]

with: \( \mathbf{s}(\mathbf{x}) \): complex signal, defined over a compact domain \( \Omega \) in \( \mathbb{R}^2 \), \( E_r(\mathbf{x}) \): open ball of radius \( r \) (the scale of observation) centered at pixel location \( \mathbf{x} \) in the signal domain \( \Omega \). The limiting behaviour of \( \mu(E_r(\mathbf{x})) \), as \( r \rightarrow 0 \), if it can be accurately computed at each pixel location \( \mathbf{x} \), provides information that generalizes appropriately, in the case of complex multiscale and turbulent signals, the classical notions of \textit{edge} or \textit{border} that forms the basic building block of many Image Processing and Pattern Recognition approaches [40]. Moreover, as it will become apparent in this work, the pointwise evaluation of singular exponents unlocks the localization of the geometric hierarchy associated to the dynamics of the flow (in the case of FDT). One important aspect of signals associated to acquisition of turbulent and complex phenomena is that the notion of transition, if properly formulated from paradigms in Statistical Physics, allows the determination of specific \textit{geometric manifolds} in the signal domain, such as the \textit{Most Singular Manifold}, that conveys key critical information about motion. This article develops on this idea, by showing that the determination of the motion field in oceanographic image sequences acquired by satellites can be done using only one \textit{frame} in the temporal sequence plus extra spatial information at lower resolution, hence eliminating the problems related to temporal luminance conservation and lowering considerably all computation difficulties coming from the presence of noise and acquisition artefacts. This achievement can be done by showing that the framework of \textit{reconstructible systems}, which comes from reconstruction formula and the notion of \textit{Most Singular Manifold}, can be combined with the idea of \textit{microcanonical energy cascade}. Reconstructible systems refer to signals that can be generated at any arbitrary precision using the most informative subset in the turbulent flow. Recent work about the \textit{Microcanonical Multiscale Formalism} [41] has shown that these ideas can be used to evaluate a first order approximation of the \textit{geostrophic stream function}. The present article provides substantial improvements on the subject, by using the microcanonical energy cascade to solve the \textit{orientation problem} of the vector field, and by generalizing the method to non-geostrophic motion.

The Microcanonical Multiscale Formalism, or MMF, is reviewed in Section 2, along with the framework of reconstructible systems. In Section 3 the notion of optimal wavelet is discussed in the natural context of the energy cascade. Section 4 develops on the main idea presented in this work: the use of the optimal wavelet in conjunction with reconstructible systems to solve the \textit{orientation problem}. Results, discussion and conclusion end the paper. In particular, the novel method is first applied on the output of a 3D oceanic circulation model to evaluate the quality of the results on synthetic data, then on real datasets coming from remote sensing.

2. The microcanonical multiscale formalism

In the microcanonical approach to multiscale properties of complex signals [37], it is assumed that, at every point \( \mathbf{x} \) in the signal domain, or equivalently in our case, at each pixel location \( \mathbf{x} \)
As shown in [42–44], this limiting behaviour is characteristic of natural images and complex signal acquisitions. The coefficient \( h(\mathbf{x}) \) is a single exponent at point \( \mathbf{x} \), and can be computed for instance using log–log estimation on wavelets projections [45,37].

To understand the fundamental concept of single exponent and its significance in the processing of complex signals, we note that the power-law behaviour of the measure \( \mu \) in Eq. (4) is a classical observation in Statistical Mechanics for intensive variables (temperature, energy dissipation etc.) around critical points. That power-law behaviour is an indication of scale invariance, such as the one observed in higher-order phase transitions [46–48].

Systems characterized by the same value of single exponents form a particular universality class: the macroscopic properties of the system are completely determined by the values of the exponents, independently of the particular system considered. Thus, in this approach to Signal Processing, every point is considered as critical, with a continuum of possible values for measuring criticality. The classical notion of edge or border in Computer Vision would correspond, in this analogy, only to first-order phase transitions. Instead of this, the consideration of a full continuum of potential values for critical points allows the definition of a generalized notion of edge much better suited to the case of turbulent and complex signals. This is illustrated in Fig. 2, where we display the computation of single exponents for the subimages of Fig. 1. The image illustrates how the single exponents quantify the transition values of pixels belonging to coherent structures.

A proper and accurate evaluation of the singularity exponents \( h(\mathbf{x}) \) leads to the consideration of the geometric arrangement of the sets

\[
\mathcal{F}_h = \{ \mathbf{x} \in \Omega | h(\mathbf{x}) = h \},
\]

arrangement which is naturally associated to the notion of information content in the signal [38,39]. Thus, in the framework of reconstructible systems, it should be possible to recover the signal from the set of its highest transition fronts. The points in the image domain which contribute to the highest transitions, i.e. those which maximize, in a statistical sense, the information contained in the signal, are precisely those points having the lowest (and negative) single exponents, that is to say the points belonging to the Most Singular Manifold \( \mathcal{F}_\infty = \{ \mathbf{x} \in \Omega | h(\mathbf{x}) = h_{\infty} \} \) with \( h_{\infty} = \min h(\mathbf{x}) \). The hierarchy of sets \( \mathcal{F}_h \) describes the geometric arrangement of information content in the signal, with a most informative content located inside the set \( \mathcal{F}_\infty \). For signals that are acquisitions of fluids under turbulent motion, like the oceans, this hierarchy is the one associated to the spectrum of exponents in the canonical formalism. Consequently, as shown in [49], there must be a reconstruction operator which enables the reconstruction of the whole signal given the restriction of its gradient vector field to the set \( \mathcal{F}_\infty \). The reconstruction \( \mathcal{G}(\mathbf{s}) \) can be explicitly determined in Fourier space [49]:

\[
\mathcal{G}(\nabla \mathbf{s})|_{\mathcal{F}_\infty}(\mathbf{f}) = \left\langle \sqrt{-1} \mathbf{f} \nabla \mathbf{s} \right\rangle|_{\mathcal{F}_\infty}.
\]

(f: frequency vector, \( \left\langle \cdot , \cdot \right\rangle \): scalar product in \( \mathbb{C}^2 \), \( \mathcal{G} \): Fourier transform). This reconstruction formula shows that the whole signal can be reconstructed given the gradient restricted to the set \( \mathcal{F}_\infty \) of most informative points. This is the framework of reconstructible systems.

### 3. Scale invariance and the energy cascade

In the motion of a fluid flow, the ratio between the largest scale \( L \) and the viscous dissipation scale \( \eta \) (corresponding to the smallest scale, where the energy is dissipated) is

\[
\frac{L}{\eta} \sim Re^{3/4}
\]

(Re: Reynolds Number) which prevents the numerical simulation of all phenomena at all intermediary scales in a regular cube of size \( L \) ([50] and references therein). The energy is injected in the system at the largest scale \( L \), entirely dissipated at viscous scale \( \eta \), and, in-between, it crosses all the intermediary scales, and is responsible for the creation of a self-similar random distribution of vortices and other complex coherent structures, which
ultimately appear as a complex arrangement of fronts and oceanographic structures in acquired images of the oceans. The intermediary structures let the energy pass through the scales, where it is terminally dissipated at scale \( \eta \). The dynamics of the flow is governed by the cascading process, which must be, in return, properly evaluated to assess any information about the motion. It turns out that a proper modelling of the cascading aspects in turbulence can be achieved through multiplicative cascades [50].

The cascading process can be retrieved easily by considering the random variables

\[
\dot{\varepsilon}_c(x) = \mu(B_c(x)) = \int_{B_c(x)} \| \nabla s(y) \| dy
\]  

so that, for two different scales \( r_1 \) and \( r_2 \), one has, for the distributions of the variables (denoting \( dP_x \) the probability law of a random variable \( X \)):

\[
dP_{r_1} = dP_{r_1} \delta_{r_1} \delta_{r_2}
\]

This equation relating the distributions of random variables \( \dot{\varepsilon}_c \) defines the random variables \( \eta_{r_1/r_2} \) which are such that \( \dot{\varepsilon}_c \) and \( \eta_{r_1/r_2} \) are independent random variables. The variables \( \eta_{r_1/r_2} \) do not depend on the scales \( r_1 \) or \( r_2 \), but only connect their ratio [51]. Eq. (9) shows that the statistics of variables \( \dot{\varepsilon}_c \) split into two parts: one, governed by \( \eta_{r_1/r_2} \) describes the properties under change of scale, and the other \( \dot{\varepsilon}_c \) takes into account the properties at a given scale of reference \( r_2 \). The multiplicative cascade property becomes mathematically apparent when considering three scales \( r_1 < r_2 < r_3 \) for one has

\[
dP_{r_1/r_3} = dP_{r_1/r_2} dP_{r_2/r_3}
\]

hence for the generation of indefinitely divisible laws.

Eqs. (9) and (10) relate only the laws of the distributions, and would not imply any corresponding relation pointwise. However, one can formally define random variables

\[
\theta_{r_1/r_2}(x) = \frac{\dot{\varepsilon}_c(x)}{\dot{\varepsilon}_c(x)}
\]

But in general, the variables \( \theta_{r_1/r_2}(x) \) defined by that equation are such that there is no independence between \( \theta_{r_1/r_2}(x) \) and \( \dot{\varepsilon}_c(x) \). To access the cascading properties of the signal pointwise (also called the microcanonical cascade), the concept of optimal wavelet can be introduced [51]. The random variables \( \dot{\varepsilon}_c(x) \) carry the multiscale properties of the signal, but their definition does not allow to extract from them the cascading properties pointwise, i.e., the microcanonical cascade. Another way of tackling this problem is to consider, for an admissible wavelet \( \psi \), and the original signal \( s \), the multiscale operator

\[
T_\psi(s|x,r) = \int s(y) \psi(\frac{x-y}{r}) dy
\]

Exactly like in Eq. (11) we define the random variables \( \zeta_{r_1/r_2}(x) \) by

\[
T_\psi(s|x,r_1) = \zeta_{r_1/r_2}(x) T_\psi(s|x,r_2)
\]

but, then, considering the wavelet \( \psi \), as a parameter, we can ask for wavelets \( \psi \) for which the variables \( \zeta_{r_1/r_2}(x) \) are independent of \( T_\psi(s|x,r_2) \). If such wavelet can be determined, it is called an optimal wavelet. If an optimal wavelet can be found, it has the fascinating potential of unlocking the signal’s microcanonical cascading properties through simple wavelet decomposition. In other words, the turbulent properties of the signal become apparent from the optimal wavelet decomposition. In particular, all the complex aspects of motion should be derived from the optimal wavelet decomposition. This is the primary idea of this work. Unfortunately, the systematic determination of an optimal wavelet, for a given signal, is an extremely difficult and unsolved problem. In this work, we will show by simply considering approximations of the optimal wavelet decomposition that we are able to enhance considerably the determination of the motion field in turbulent fluid flow acquisitions of the oceans. This is described in the next sections.

4. Descending motion information through an optimal wavelet

In [41,52], authors presented a new method for the evaluation of the (geostrophic) oceanic motion field from a temporal sequence of Sea Surface Temperature satellite images. The method consists in using the framework of reconstructible systems (recalled in Section 2) to generate a first approximation of the stream function associated to oceanic flow in complex signals of Sea Surface Temperature (SST) satellite acquisitions. Once the stream function is computed, a vector field associated to oceanic motion is automatically generated by computing the gradient of the approximated stream function and turning it by a \( \pi/2 \) angle. The method is remarkable from the following points of views:

1. it makes use of a single temporal acquisition in an SST image sequence acquired by satellite, as opposed to all other types of algorithms based on luminance conservation described in Section 1,
2. it is robust and consistent w.r.t. acquisition artefacts and missing data.

However, the algorithm suffers from a main limitation: the orientation of the resulting vector field is not determined properly. This is the orientation problem. The drawback comes from the fact that the gradient is rotated by a fixed right angle. In other words, only the straight line that supports the motion is determined, but the direction on that line remains unknown. We show that using an approximation of the microcanonical cascade does help fix the orientation problem.

The idea is to propagate a correct orientation through the microcanonical cascade, in the form of scalar products with vector data acquired at the same time but with lower spatial resolution. The method is described in the following subsections.

4.1. Approximation of the microcanonical cascade

The exact determination of the optimal wavelet for a given dataset is difficult in general, and there is no systematic algorithm up to this date [53]. Consequently, we will approximate the optimal wavelet. In our experiments, we took sub-images of MODIS SST images (see Fig. 1) and made experiments with Haar, compact-supported Daubechies and order 3 Battle-Lemarie wavelets [53].

In the wavelet analysis of 2D signals, the multiresolution analysis can be seen in the form of a “genealogy tree”: wavelet coefficients in one of the three orientations (horizontal, vertical and diagonal details) and at a given scale resolution are members of a same “family”: a “son” coefficient has two cousins, corresponding to the other two orientations (at the same scale and spatial localization). A “son” coefficient has a “father” at lower scale and two “uncles” (two other orientations) and a “father” has four “sons” (see [45,53]). The three orientations correspond to: the signal’s approximations, the horizontal and vertical details associated to the multiresolution analysis (approximation subspaces and their orthogonal complements). We
subscript the wavelet coefficients by their “genealogy” in the tree: \( x_f \) for the father \( x_s \) for the son etc.

Persistence along the scales implies a relation of the form

\[
x_s = \eta_1 x_f + \eta_2
\]  

(14)

with \( \eta_1 \) and \( \eta_2 \) being random variables independent of \( x_s \) and \( x_f \) and also independent of each other. For the optimal wavelet, one would have \( x_s = \eta_1 x_f \) [53], which implies \( \log|x_s| = \log_2|\eta_1| + \log_2|x_f| \).

So, a test for optimality consists in examining the conditional histograms \( E(\log_2|\eta_1|, \log_2|x_f|) \) for the different types of coefficients in the wavelet-based wavelet analysis of our dataset. In this experiment, we took a set of 900 subimages (chosen for having few missing data) in the Modis dataset of all the years 2006–2008 (Fig. 1 shows the acquisition of August 2, 2007, however the 900 subimages are taken in the whole years of 2006–2008 of Modis data).

Fig. 3 shows that order 3 Battle-Lemarié wavelets with 41 central coefficients provide a good approximation of the optimal wavelet: the functional dependencies are almost linear both for the approximation and orthogonal complements, with a reasonable loss of linearity in the orthogonal complements. Consequently, we take the order 3 Battle-Lemarié wavelets with 41 central coefficients as our approximation of the optimal wavelet.

Note that the exact optimal wavelet is fully dependent on a given signal.

4.2. The algorithm

Our goal is to compute the motion field \( \dot{u}(x) \) of ocean dynamics at the high resolution of MODIS SST (Sea Surface Temperature) data (spatial resolution: 4 km) using one single image (as opposed to the various optical flow and estimation techniques described in Section 1, which make use of several temporal occurrences around the frame at time \( f \)), and which resolves the orientation problem by using auxiliary data at larger resolution.

The key idea of the method lies in Eq. (13) which can be used by replacing the signal \( s \) by any signal representing a scalar quantity subject to the microcanonical cascade. In particular, we use the microcanonical cascading of scalar products to resolve the orientation problem.

We make use of the motion field available at much larger resolution (1/4 longitudinal degrees), and acquired at a near temporal occurrence, given from altimetric data [54]. Let \( \dot{w}(x,r_2) \) be this vector field, available at scale \( r_2 > r_1 \), where \( r_1 \) is the high resolution of MODIS SST data at which we want to compute the motion field. Let \( s(x,r_1) \) be a signal acquired at high resolution \( r_1 \); in our experiments, \( s(x,r_1) \) can be the SST (Sea Surface Temperature) itself or another signal computed from it: for instance the values of the singularity exponents or the gradient’s norms of the singularity exponents. We perform multiscale analysis on this signal to compute the wavelet projections \( T_{\psi}(s(x,r_1)) \) with order 3 Battle-Lemarié approximated optimal wavelet \( \psi \). We obtain then a projected signal:

\[
T_{\psi}(s(x,r_1)) = \sum_{n \in \mathbb{Z}} \langle s(x,r_1), \psi_{j,n} \rangle \psi_{j,n}
\]  

(15)

(\( \psi_{j,n} \): scaled and translated wavelet \( \psi \) at resolution \( r_j \)). When resolution \( r_j \) is reached, we compute the singular exponents \( \eta_j(x) \) at resolution \( r_j \), from which we determine the Most Singular Manifold at resolution \( r_j \) \( \mathcal{F}^{r_j} \). Using the reconstruction formula (6), an approximated stream function at resolution \( r_j \) can be generated using the vector field \( v^j(x,r_j) \) which is just the normal field of the submanifold \( \mathcal{F}^{r_j} \). In other words, \( v^j(x,r_j) \) is the unitary vector field perpendicular to \( \mathcal{F}^{r_j} \). The approximated stream function at resolution \( r_j \) is given in Fourier space by the formula:

\[
\widehat{\mathcal{N}}_j(f) = \frac{\sqrt{-i f \mathcal{N}(\langle x,r_j \rangle, r_j^2)}}{|f|^2}.
\]  

(16)

The vector field \( \mathcal{N}_j \) is defined at altimetry resolution \( r_j \) and is a first order approximation of the stream function at that scale. Then, we can compute scalar products

\[
\langle \dot{w}(x,r_2), \mathcal{N}(x,r_2) \rangle
\]  

(17)

with the given altimetry vector field \( \dot{w}(x,r_2) \) and test for the negativity of these scalar products: whenever such a scalar product is negative at a point \( x \), the vector is changed in its opposite \( -\mathcal{N}_j \). Let \( \mathcal{M}(x,r_2) \) the new vector field obtained this way. Then, a new oriented stream function is generated in Fourier space using the reconstruction formula:

\[
\widehat{\mathcal{N}}_j(f) = \frac{\sqrt{-i f \mathcal{N}(\langle x,r_2 \rangle, r_2^2)}}{|f|^2}.
\]  

(18)

That low-resolution stream-function is then back-projected through the approximated optimal wavelet to obtain an oriented stream function at the higher SST resolution \( r_1 \) (Eq. (15)): \( \mathcal{C}_r(x) \). As a result, we take as unitary oriented motion field at SST resolution \( r_1 \), the vector field

\[
\dot{u}(x) = \left( \nabla \mathcal{C}_r(x) \right) \perp
\]  

(19)

perpendicular to \( \nabla \mathcal{C}_r(x) \).

The basic idea behinds this algorithm (“use optimal cascading to decrease spatial resolution, use motion information available at lower resolution, then go and carry that information back to higher spatial resolution using the cascade to perform corrections”) can be declined in several ways. First, the choice of the cascade itself may be subject to experimentation (on can use an optimal wavelet associated to the signal itself or an optimal wavelet associated to the singularity exponents’ gradient norms for instance). Second, one can perform back-cascading on the stream function as presented above, or on a new signal generated at lower spatial resolution, and defined by the scalar product of the altimetry vectors \( \dot{w}(x,r_2) \) and the gradient to the stream function; once that scalar product is back-projected at higher resolution, use its sign to change the orientation of a stream function computed directly at high resolution (or re-direct exactly the singularity exponents’ gradients – a very good solution whenever the spatial resolution makes the geostrophic approximation no longer valid -). We found that latter variant more accurate in our experiments.

In the following two sections, we apply the methods on two types of data: first, on the output of a 3D oceanic circulation model to evaluate the performance of the method, then on real remotely sensed data.

5. Results on synthetic ROMS data

5.1. Model description

The hydrodynamic model is the Regional Ocean Modelling System (ROMS). The reader is referred to [55,56] for a more complete description of the model. It simulates the salient features of the large-scale circulation patterns as well as the coastal upwelling features of the marine ecosystem. It is a split-explicit and free-surface model which considers the Boussinesq and hydrostatic assumptions when solving the primitive equations. This model has been adapted to the Benguela upwelling subregion [57,58]. We are using here the most recent configuration covering the northern and southern Benguela system of Veitch et al. [59]. The model is discretized in the vertical on a sigma or topography—following stretched coordinate system. The
5.2. Experiments

Our experiments with ROMS consisted in evaluating the results on a set of 10 samples. For each of the 10 samples, we have the following data:

- a high spatial resolution (pixel size $r_1$ corresponding to 4 kms on Earth) computed SST simulation,
- the motion field $w(x, r_1)$ computed by ROMS at resolution $r_1$,
- a lower spatial resolution (pixel size $r_2$ corresponding to 28 kms on Earth) computed SST simulation,
- the motion field $w(x, r_2)$ computed by ROMS at resolution $r_2$.

The simulation employs the 2-way embedding capability of ROMS, which is designed such that the output from a lower resolution “parent” domain provides boundary conditions for the higher resolution “child” domain nested within it and the “child” domain in turn feeds the parent domain. Both the parent and child grids have 32 sigma-levels stretched so that near-surface resolution increases. We use a set of 10 outputs both at high (pixel size: 4 kms) and low (pixel size: 28 kms) resolution. See Fig. 4 for a sample output of simulated passive scalar, associated motion computed with ROMS and singularity exponents. Of course, the output of the model features a computed velocity field that we use to assess the method presented in the previous sections.

Fig. 3. Conditional histograms $E(\log_2|a_s|/\log_2|a_f|)$ made with 900 subimages extracted from a 2006–2008 MODIS SST dataset. The horizontal axis corresponds to the father and the vertical to the sons. The wavelet used for this experiment is the order 3 Battle-Lemarié wavelet with 41 central coefficients. Top: approximation coefficients are depicted on the left image, horizontal details (orthogonal complements) are depicted on the right image. Bottom: vertical coefficients are depicted on the left image, diagonal details are depicted on the right image (both correspond to orthogonal complements in the multiresolution analysis).
The experiment consisted in computing an optimal wavelet approximation of the cascade associated to the norm of the singularity exponents' gradients, starting at resolution $r_1$ and reaching resolution $r_2$. At resolution $r_2$, a new signal is generated, defined as the scalar product between available data $w(x, r_2) / \|w(x, r_2)\|$ from ROMS simulation and the singularity exponents' gradients computed at resolution $r_2$ from the approximated optimal cascading process. The scalar product is then projected back to resolution $r_1$ using the same approximated optimal wavelet decomposition and the sign of this reconstructed...
The scalar product is used to invert the orientation of a stream function computed at resolution $r_1$. The results are summarized in the table shown in Fig. 5.

### 5.3. Discussion

As the table shows, the optimal wavelet method produces much better results than the stream function method, at least with synthetic data. On the average, more than 30% points still are badly oriented. In Fig. 6 we show the resulting unitary field obtained by our optimal wavelet method, called here **OW Algorithm**, on one image in the ROMS set, its comparison with the field produced by ROMS, and a map of the scalar product between the two normalized fields.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Average</th>
<th>Best</th>
<th>Std. dev. (scalar product)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OW Algorithm</td>
<td>62.66%</td>
<td>64.75%</td>
<td>1.24</td>
</tr>
<tr>
<td>SF Algorithm</td>
<td>49%</td>
<td>50.40%</td>
<td>5.46</td>
</tr>
</tbody>
</table>

**Fig. 5.** Quantitative results for the evaluation of the algorithm presented in this paper on a 10-sample ROMS simulation output. **OW Algorithm** stands for the “stream function” algorithm presented in [41,52], and which consists in assigning an arbitrary orientation for the stream function, without any cascade decomposition. “Average” and “Best” refer to the percentage of vectors correctly oriented (i.e. having a positive scalar product with vectors obtained from ROMS at resolution $r_1$), both in the average for the set of 10 samples, and for the best result in that set. “Standard deviation” refers to the standard deviation of the scalar product in the subset of correctly oriented vectors. Besides an overall augmentation of correctly oriented vectors obtained with the **OW Algorithm**, both in the average and in the best case, a noticeable result is the lower standard deviation obtained with the **OW Algorithm**, which indicates a more stable vector field, at least on synthetic ROMS data.

**Fig. 6.** Visual inspection of the vector field produced by the **OW Algorithm**. **Top:** on the left, we show the field produced by the ROMS modelling software, and on the right the unitary field computed by the **OW algorithm**. **Bottom:** map of the scalar product between the two (unitary) fields. The blue points, which denote a reversed orientation, are following coherent structures.
We think that the existence of remaining points with a reversed orientation comes, in some respect, from the fact that we use an approximation of the optimal wavelet, and not the optimal wavelet itself. Since the determination of an exact optimal wavelet is an extremely difficult problem, our research will now focus on such a determination.

Fig. 7. Top: sample of a MSLA (mean sea level anomaly) dataset used in the algorithm as an input for the velocity field at lower spatial resolution. Spatial resolution is about 28 kms. Pixel values denote the deviation of the sea level surface from a standard medium value. From the MSLA, a geostrophic vector field can be generated directly. Bottom: geostrophic vector fields computed from MSLA in the black squares of top picture, magnified.
6. Results on real data

We now show the results obtained by applying the previous algorithm on images extracted from the MODIS Level 3b dataset acquired in 2007. In this dataset, there is one image per day (an example is shown in Fig. 1). Pixel size is approximately 4 kms. The surface velocity field [54] used, at the much lower spatial resolution of $1/4$, comes from a combination of wind-driven Ekman currents, at 15 m depth, derived from QuikSCAT daily wind estimates, and weekly geostrophic currents calculated from Sea Surface Heights (SSH) which are obtained by weekly Mapped Sea Level Anomalies (MSLA) associated with a Mean Dynamics Topography (MDT). These velocity fields (Fig. 7) data correspond to a pixel size of $\sim 28$ kms and daily periodicity which are computed by adding a temporal linear interpolation of weekly geostrophic currents with a daily Ekman currents.

In Fig. 8 we display the resulting unitary velocity field for the left subimage of Fig. 1. In the background the values of the singularity exponents are shown in grey-level values, hence evidencing the proper delimitation of complex coherent structures achieved by the values of the exponents. Along the

Fig. 8. The resulting unitary motion vector field for the left subimage of Fig. 1. Vectors are depicted in red, and they are drawn over the background image of the singular exponents, shown in grey-level values. For better visualization, the original subimage has been divided into four equal parts shown in this picture. The original subimage of Fig. 1 was specifically chosen for its abundance of complex currents, fronts, vortices and other oceanic structures, and also for its highly turbulent character, evidenced by a simple computation of Lyapunov exponents. The resulting vector field follows particularly well all the complex fronts and structures, with a remarkably well matching of the orientations even for the smallest vortices.
fronts and coherent structures, the resulting velocity field follows the streamlines of the approximated stream function, hence showing, at least experimentally, the advection of the singularity exponents by the oceanic flow. Notice also the proper delimitation of vortices achieved by the resulting field.

The resulting vector field is also shown in Fig. 9 for the other subimage of Fig. 1, of even smaller size. We increased the magnification in this picture to show the resulting vector field at higher spatial scales. We also display the input data consisting of the vector field derived from altimetry at spatial resolution of 28 kms.

7. Conclusion

In this paper, we propose an original and better determination of an unitary oriented vector field describing ocean dynamics on high resolution satellite data using one single temporal frame and initial data given in the form of altimetry velocity field obtained at lower spatial resolution. Because it only uses one temporal frame, the method eliminates the multiple drawbacks inherent to methods that make use of several temporal frames, and does not hinge on conservation laws that can be subject to criticism (both from a physical point of view and also from all difficulties bound to the acquisition process) when dealing with complex signals displaying strong turbulent properties.

The determination of the unitary motion field makes uses of recent advances in the multiscale analysis of complexity in turbulent signals, in which the precise numerical computation of a singularity exponent around any point allows the determination of the geometric subsets both associated to transition fronts and statistical content. The significance of such geometric subsets, like the Most Singular Manifold, is asserted in the framework of reconstructible systems, which develops reconstruction formulae based on the statistical content located in these sets.

The main idea presented in this paper is in the use of an optimal wavelet, which provides a wavelet decomposition closely related to the multiscale energy cascade observed in fully developed turbulence. Since the computation of the optimal wavelet remains a difficult and open problem, we show that the consideration of an approximated optimal wavelet enhances to a large extent previous results. The optimal wavelet decomposition is used in conjunction with specific reconstruction formula in the framework of reconstructible systems to generate a motion field at lower resolution. That generated motion field is re-oriented by taking scalar products with an available motion field derived from an altimetric dataset acquired over the same temporal period, but at much lower spatial resolution. The re-oriented vector field is then back-projected through the wavelet decomposition to compute an oriented stream function at much higher spatial resolution. The systematic use of the Microcanonical Multiscale Formalism shows its ability to make use of information acquired at different spatial resolutions, a very interesting and promising approach in the field of remote sensing.

The methodology introduced in this work opens the way to a vast domain of research: the determination of the exact optimal wavelet from the signal data, the propagation of norm information through the cascade to obtain a valid normed and oriented field, the determination of a dense motion field even in the presence of large missing data. Moreover, there is a well defined generalization to the case of non-geostrophic motion, like the Ekman component of the oceanic motion field. These research topics are currently being investigated by the authors.

Acknowledgments

This work is funded by the NASA-EUMETSAT-CNES Hiresubcolor contract (title: Hiresubcolor: Multiscale methods for the evaluation of high resolution ocean surface velocities and subsurface dynamics from ocean color, SST and altimetry). Hiresubcolor is a participation to the Ocean Surface Topography Science Team (OST-ST). The authors also thank the anonymous reviewers for their suggestions.

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