Multiscale Methods in Signal Processing for Adaptive Optics

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Acknowledgement: This work has been done in collaboration with ONERA.
1. Motivation

2. Modelling Turbulence

3. Optimal Inference with Singularity Exponents

4. Phase Reconstruction with Singularity Exponents

5. Results and Conclusion
Problem Statement

- Earth’s atmosphere: imperfect media to view spatial objects.
- Refractive index variations interfere with light propagation.
- Results in distortion of planar wavefront.
- Distortion due to non-constant distribution of wavefront phase.
- Refractive blurring of images.
Current Solution: Adaptive Optics

- Consists of 3 principal components.

- **Wavefront sensor (WFS)** measures wavefront phase distortion.

- **Controller**: generates corrected (control) signals based on WFS measurements.

- **Deformable mirror**, driven by the control signals, adjust itself to incident wavefront.

- Removes (approximately) phase distortion from incident wavefront.

Globular cluster Omega Centauri.
- **Top**: without AO correction,
- **Bottom**: with AO correction.

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Signal Processing for Adaptive Optics
The Shack-Hartmann (SH) WFS

- Consists of NxN lenses in conjugate pupil plane.
- Each lens forms an image of source on a detector.
- Distorted wavefront: Image shifted from reference position.
- Shift in image centroid $\propto$ average wavefront slope over sub-aperture.
- Measured as phase gradients averaged over sub-aperture area.

Part of detector corresponding to a single lens (sub-aperture area.)
Wavefront Reconstruction in Controller

- A wavefront reconstructor recreates wavefront from WFS measurements.

**Aim**

- Estimate wavefront phase values from gradient measurements.

**Linear Model:** \( g = \Gamma \Phi + n \).
  - \( g \): vector of measurements (SH slopes).
  - \( \Phi \): vector of unknowns (wavefront phase).
  - \( \Gamma \): differential operator, \( n \): measurement noise.
  - One searches for \( \hat{\Phi} \) that minimizes \( \hat{\Phi} = \arg\min_{\Phi} \| \Gamma \Phi - g \|^2_2 \).

**Classical estimate**

- **Least squares solution:** \( \hat{\Phi} = (\Gamma^T \Gamma)^{-1} \Gamma^T g \).
Objective

Goal:
- Present alternate approach to estimate wavefront phase in AO.
- Generate high-resolution phase from low-resolution gradients.
- Robust to sensor noise present in gradient measurements.

Strategy:
- Optimal inference across scales of a turbulent phase.

Methodology:
- Microcanonical Multiscale Formalism (MMF).
Modelling Atmospheric Turbulence
Kolmogorov’s theory on energy cascades

- Characterized by random vortices (turbulent eddies).
- Ranges from hundreds of meters to a few millimeters.
- Energy transmitted successively from higher size (or scale $L$) eddies to increasingly lower size (scale $r$) eddies.

Important Inference

- In fully developed turbulence (FDT): area between two scales ($0 < r < L$) called the inertial subrange.
- Domain knowledge important in describing wavefront distortions.
Kolmogorov’s theory cntd...

- Process of energy transfer between $r$ and $L$:
  \[ |T_r s| \sim \eta_r/L |T_L s| \]
  - where $\eta_r/L = [r/L]^{\delta}$, independent of $T_L$.
  - $T_r$: local dissipation of energy of $s$ around radius $r$.

**Important Inference**

- Holds in distributional case, not pointwise: $T_r s(\vec{x}) \neq \eta_r/L T_L s(\vec{x})$.
- $\eta_r/L$ indefinitely divisible to realize cascade process.

**Important Inference on $\eta$**

- Describes systems under scale changes.
- Pointwise estimate: insight in describing multiplicative cascade process.
An operator $T_r$ that can extract information pointwise.

Define an operator $T_r = T_\Psi$ : wavelet projection of $s$ on $\Psi$ at position $\vec{x}$ and scale $r$.

$T_\Psi s(\vec{x}, r)$ defines a random variable $\zeta_{r/L}(\vec{x})$ such that:

$$T_\Psi s(\vec{x}, r) = \zeta_{r/L}(\vec{x}) T_\Psi s(\vec{x}, L)$$

Important conclusions on wavelet $\Psi$

- $\Psi$ (if determined) will make $\zeta_{r/L}(\vec{x})$ independent of $T_\Psi s(\vec{x}, L)$.
- Such a wavelet is called an *optimal wavelet*.
- Optimality of a wavelet = degree of independence of $\zeta_{r/L}(\vec{x})$ vs $T_\Psi s(\vec{x}, L)$. 
Important Inferences

- Multiresolution analysis associated to an optimal wavelet = optimal information inference across scales.

Problem
- Difficult to compute $\Psi$ for a turbulent acquisition.
- Unsolved and open problem.
- Previous approaches only gives a sub-optimal wavelet [Pont et al, 2011].

Alternate Approach
- Proper models to explore multiplicative cascade process.
- Realize multiscale hierarchy that exists in multiplicative cascade.
- Ensure optimal inference across scales.
Model

- Multiscale analysis of turbulent signals using multifractals.
- Turbulent flows well-defined by multifractal hierarchy.
- Scale-independent system: explores self-similarity in a signal.
- Consists of multiple fractal components: organization related to multiscale hierarchy.
- Key quantity: collection of all fractal dimensions.
- First instances in Kolmogorov’s theory on FDT.

Framework used

- Microcanonical Multiscale Formalism (MMF).
Microcanonical Multiscale Formalism
Canonical Multiscale Formalism: CMF

\[
\langle |T_r s|^p \rangle = \alpha_p r^{\tau_p} + o(r^{\tau_p}) \text{ where } r \to 0
\]

\( T_r \): given family of functions.
\( \langle \cdot \rangle \): average over ensemble of signals, \( s \) belongs to.
\( \alpha_p \) depends on \( T_r \).

- For 2 scales \( r \) and \( L \) [A. Turiel et al, 2008]:
  \[
  \frac{\langle |T_r s|^p \rangle}{\langle |T_L s|^p \rangle} = \left( \frac{r}{L} \right)^{\tau_p}
  \]

  \( \tau_p \): scaling exponents.

Important Inference

- \( \tau_p \) as a function of \( p \) is a convex curve [U. Frisch, 1995].
- Important characteristic, existence of multiscale hierarchy [U. Frisch, 1995].
Inferences on $\tau_p$

- Any function solving $\tau_p$ leads to same distribution of $\eta_r/L$ [A. Arneodo, 1995].
- Knowledge of $\tau_p = \text{insight into cascade formation process.}$

**Problem**

- Highly data demanding, computationally expensive.
- No access to geometrical arrangement of fractal components, $\tau_p$ is only a global characterization.

**Solution**

- Averages of different points $\bar{x}$ within same realization.
Microcanonical Approach to Multifractals: MMF

\[ T_r s(\vec{x}) = \alpha(\vec{x}) r^{h(\vec{x})} + o(r^{h(\vec{x})}) \quad r \to 0 \]

\( T_r \) : appropriate scale-dependent functional.
\( h(\vec{x}) \): the singularity exponent (SE) of \( \vec{x} \).

- Log-domain representation : \( \log T_r s(\vec{x}) = h(\vec{x}) \log r + \log \alpha(\vec{x}) \)
- Estimation of \( h(\vec{x}) \) from \( T_r \): log-log regression.

Comments
- Gives access to geometrical arrangement of fractal components \( \mathcal{F}_h \):
  \[ \mathcal{F}_h = \{ \vec{x} : h(\vec{x}) = h \} \]
**Choice of the Functional \( T_r \)**

- **Gradient-Modulus measure** [A. Turiel et al]:
  \[
  T_r s(\vec{x}) = \int_{B_r(\vec{x})} d(\vec{y}) \| \nabla s \| (\vec{y})
  \]

- **Wavelet projection** [A. Turiel et al]:
  \[
  T_\psi s(\vec{x}, r) = \int \| \nabla s \| (\vec{y}) \psi(\frac{\vec{x} - \vec{y}}{r}) d\vec{y}
  \]

- Compute SE over finest scale (resolution) \( r_0 \):
  \[
  h(\vec{x}) = \frac{\log(\langle T_\psi s(., r_0) \rangle)}{\log r_0}
  \]

- \( \langle T_\psi s(., r_0) \rangle \) = average of the measure.

---

**Top:** Experimental phase screen.  
**Bottom:** Singularity exponents.

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Signal Processing for Adaptive Optics
Singularity Spectrum

- Collection of all the fractal dimensions $D(h) = \dim_{H} F_h$.
- $D(h)$ represented as a function of $h$.

Reduced singularity spectra $D(h) - d$ for the experimental phase screen: **Left**: at the finest scale, **Right**: at 3 different scales.

Inference

- Convex shape: presence of multiscale hierarchy in phase data.
Important notes: Singularity Exponents

- Provides rich framework for describing multiscale hierarchy in turbulent signals.
- Encode transitions present in a turbulent signal (generalizes edges in natural images).
  - Edge represents basic multiscale features in a signal.

Optimal Inference Idea

- **Goal**: To show SE carry the relevant multiscale features of a signal.
- Sufficient candidate for optimal inference.

Experimental justification

- Notion of edge, well adapted for turbulent signals, coherent across scales.
- Edge information sufficient to reconstruct the whole signal.
Step 1

Edge detection & Edge consistency across scales
Edge detection & Singularity Exponents

- Fractal set associated to the smallest values of $h(\bar{x})$

$$\mathcal{F}_\infty = \{\bar{x} : h(\bar{x}) = h_\infty = \min(h(\bar{x}))\}$$
Edge consistency

- Image used: Excerpt of sea surface temperature (SST) image.
- Corresponds to acquisition of turbulent phenomenon.
- Multiscale representation with 2 models:
  - Dyadic downsampling.
  - Linear scale-space representation [T. Lindeberg, 1998].
Edge consistency: Dyadic downsampling

- **Image**: SST (turbulent phenomenon).
- **Approximation images**: Haar DWT.
- **Edge pixel density same (approx)**.

**Inference**

- **Canny**: Difficult to match edge pixels.
- **MSM**: Transition well recorded, outperforms Mallat-Zhong.
Edge consistency: Lindeberg representation

- Linear scale-space representation, scale parameter $t > 0$.

**Inference**

- Lindeberg detector: Difficult to match edge pixels.
- MSM: Consistent edge pixels across scales.
Image reconstruction from edge representation
Reconstruction technique: $R_{\text{msm}}$

- We turn to the kernel introduced in [Turiel et al, 2002]*, and derive a parallel reconstruction kernel.

- A universal reconstruction kernel $\mathbf{\bar{g}}; s(\mathbf{x}) = \mathbf{\bar{g}} \ast \nabla_{\infty} s(\mathbf{x})$
  - $\nabla_{\infty} s(\mathbf{x}) = \text{gradient restricted to MSM}$.

- Estimating $\mathbf{\bar{g}}$: Given vector field $\mathbf{\bar{f}}$, seek $s(\mathbf{x})$ such that $\nabla s$ is close to $\mathbf{\bar{f}}$.

- We minimize $\arg \min_s \int \int (\nabla s(\mathbf{x}) - \mathbf{\bar{f}}(\mathbf{x}))^2 \; d\mathbf{x}$.

- Solving Euler-Lagrange: $\text{div}(\nabla s)(\mathbf{x}) = \text{div}(\mathbf{\bar{f}})(\mathbf{x})$.

Projection into Fourier basis gives

\[ \hat{s}(\vec{\omega}) = -i \frac{\omega_x \hat{f}_x(\vec{\omega}) + \omega_y \hat{f}_y(\vec{\omega})}{\omega_x^2 + \omega_y^2} \]

This suggests the kernel as \( \hat{g}(\vec{\omega}) = \frac{\vec{\omega}}{i\|\vec{\omega}\|^2} \).

Final expression of the reconstruction formula over the MSM in the Fourier domain

\[ \hat{s}(\vec{\omega}) = \frac{\langle \vec{\omega} | \hat{\nabla}_\infty s(\vec{\omega}) \rangle}{i\|\vec{\omega}\|^2} \]

Fourier inversion gives the reconstructed image.
Results: Reconstruction over different edges

- Reconstruction over turbulent data: Perturbated phase (row 1) & SST data (row 2).

<table>
<thead>
<tr>
<th>Original</th>
<th>MSM</th>
<th>NLFS*</th>
<th>Canny</th>
<th>LoG</th>
<th>Sobel</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Original Image" /></td>
<td><img src="image2" alt="MSM Image" /></td>
<td><img src="image3" alt="NLFS* Image" /></td>
<td><img src="image4" alt="Canny Image" /></td>
<td><img src="image5" alt="LoG Image" /></td>
<td><img src="image6" alt="Sobel Image" /></td>
</tr>
<tr>
<td>SSIM = 1</td>
<td>SSIM = 0.9986</td>
<td>SSIM = 0.9983</td>
<td>SSIM = 0.9293</td>
<td>SSIM = 0.9326</td>
<td>SSIM = 0.9886</td>
</tr>
<tr>
<td><img src="image7" alt="Reconstruction Image" /></td>
<td><img src="image8" alt="Reconstruction Image" /></td>
<td><img src="image9" alt="Reconstruction Image" /></td>
<td><img src="image10" alt="Reconstruction Image" /></td>
<td><img src="image11" alt="Reconstruction Image" /></td>
<td><img src="image12" alt="Reconstruction Image" /></td>
</tr>
<tr>
<td>SSIM = 1</td>
<td>SSIM = 0.9988</td>
<td>SSIM = 0.9957</td>
<td>SSIM = 0.9182</td>
<td>SSIM = 0.9202</td>
<td>SSIM = 0.9665</td>
</tr>
</tbody>
</table>

Inference

- Better reconstruction over MSM points.

Results: Reconstruction under noise

Row 1 = SNR 26 dB & Row 2 = SNR 14 dB.

<table>
<thead>
<tr>
<th>Image</th>
<th>MSM</th>
<th>NLFS</th>
<th>Canny</th>
<th>LoG</th>
<th>Sobel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSNR , MSE</td>
<td>PSNR , MSE</td>
<td>PSNR , MSE</td>
<td>PSNR , MSE</td>
<td>PSNR , MSE</td>
</tr>
<tr>
<td>Phase</td>
<td>23.30 , 0.0230</td>
<td>22.77 , 0.0261</td>
<td>7.69 , 0.8889</td>
<td>6.55 , 1.0722</td>
<td>13.07 , 0.2603</td>
</tr>
<tr>
<td></td>
<td>19.35 , 0.0884</td>
<td>19.02 , 0.0883</td>
<td>7.17 , 0.9437</td>
<td>6.00 , 1.2843</td>
<td>10.65 , 0.5678</td>
</tr>
<tr>
<td>SST</td>
<td>23.47 , 0.0159</td>
<td>18.02 , 0.0427</td>
<td>5.22 , 0.8383</td>
<td>5.41 , 0.7707</td>
<td>10.11 , 0.3280</td>
</tr>
<tr>
<td></td>
<td>19.21 , 0.0822</td>
<td>17.85 , 0.0979</td>
<td>5.09 , 0.9554</td>
<td>4.96 , 0.8102</td>
<td>9.74 , 0.4179</td>
</tr>
<tr>
<td>imk01305</td>
<td>29.23 , 0.0627</td>
<td>27.53 , 0.0951</td>
<td>16.82 , 1.1749</td>
<td>17.57 , 0.9494</td>
<td>23.83 , 0.2232</td>
</tr>
<tr>
<td></td>
<td>26.21 , 0.1247</td>
<td>25.23 , 0.1595</td>
<td>15.98 , 1.3719</td>
<td>17.45 , 0.9709</td>
<td>22.34 , 0.3155</td>
</tr>
</tbody>
</table>

Inference

Better reconstruction over MSM points in the case of noise.
Results: Performance of $R_{msm}$

Inference

- High quality reconstruction from MSM, $R_{msm}$ better over others.
- Reconstruction over MSM with same pixel density.

*A. Agrawal et al: What is the Range of Surface Reconstructions from a Gradient Field?, ECCV 2006*
**Results: Reconstructors + Edge detectors**

| Inference | Combination of MSM and $R_{\text{msm}}$ gives the best reconstruction. |

<table>
<thead>
<tr>
<th>Method</th>
<th>R$_{\text{msm}}$</th>
<th>Poisson solver</th>
<th>M-estimator</th>
<th>Diffusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSM</td>
<td>SSIM = 0.9942</td>
<td>SSIM = 0.9935</td>
<td>SSIM = 0.9936</td>
<td>SSIM = 0.9908</td>
</tr>
<tr>
<td>NLFS</td>
<td>SSIM = 0.9938</td>
<td>SSIM = 0.9915</td>
<td>SSIM = 0.9921</td>
<td>SSIM = 0.9577</td>
</tr>
<tr>
<td>Canny</td>
<td>SSIM = 0.9638</td>
<td>SSIM = 0.9578</td>
<td>SSIM = 0.9526</td>
<td>SSIM = 0.9445</td>
</tr>
</tbody>
</table>
Important conclusions

- SE retain the multiscale features of a turbulent signal across scales.
- Information retained is sufficient in terms of reconstructibility of the whole signal.

Conclusion

- We prove experimentally that SE carries the important multiscale information of a signal.

Next Approach

- Multiresolution analysis on SE can provide optimal inference across scales.
- Experimental validation: Application to phase reconstruction in AO.
Wavelet for MRA: Third order Battle-Lémarie wavelet

- Wavelet coefficients dependancy between two scales [Pottier et al]
  \[ \alpha_c = \eta_1 \alpha_p + \eta_2 \]
  - \( \alpha_c \) = coefficient at finer scale, \( \alpha_p \) = coefficient at coarser scale, \( \eta_1, \eta_2 \) = random variables.
- Optimal wavelet case: \( \alpha_c = \eta \alpha_p \)
- Log domain representation: \( \ln |\alpha_c| = \ln |\eta| + \ln |\alpha_p| \)

Diagonal | Horizontal | Vertical

- Vertical axis: \( \ln |\alpha_c| \); Horizontal axis: \( \ln |\alpha_p| \).
Wavefront Phase Reconstruction: Our Approach
Reconstruction Approach

Objective

- Reconstructing high-resolution phase from low-resolution gradient measurements.

- Two-step process: Analysis & Synthesis.
Reconstruction Approach

Analysis
- MRA on SE (computed on high-resolution phase).
- Extract details for every level.
- Repeat till approximation image size = size of low-resolution gradients.

Synthesis
- Replace approximation with low-resolution phase gradients.
- Reconstruct to high-resolution using intermediate details.
- Estimate phase from high-resolution reconstructed gradients.
Experimental data

- Provided with 1000 instances of high-resolution (HR) phase from ONERA.
- Gradients computed over the HR phase data.
- Averaged over windows of size $8 \times 8$ pixels to produce low-resolution (LR) gradients.

![HR phase](image)
![LR x gradients](image)
![LR y gradients](image)

128 $\times$ 128 pixels  
16 $\times$ 16 pixels  
16 $\times$ 16 pixels
Results

Experimental Results
Results: Phase

- Original phase
- Reconstructed phase (using exponents)
- Reconstructed phase (using phase image)

MSE=0.3731, PSNR=28.42 dB

- Reconstruction over the low-resolution 16 × 16 pixels gradients.

LR x gradients

LR y gradients
Motivation
Modelling Turbulence
Optimal Inference with Singularity Exponents
Phase Reconstruction with Singularity Exponents
Results and Conclusion

Results: PSF

Point spread function (PSF)

$$PSF : k[\phi](x, y) = |\mathcal{F}^{-1}\{P(x, y)e^{i\phi(x,y)}\}|^2$$

- $\phi$: perturbated phase, $P(x, y)$: telescope pupil function.
Results: Modulus of the OTF

MTF : Xcut

MTF : Ycut

MTF

Optical Transfer Function (OTF) = Fourier transform of PSF.
Modulation Transfer Function (MTF) = Modulus of OTF.
Results: Performance under noise - Phase

SNR = 40 dB
PSNR = 28.03 dB

SNR = 20 dB
PSNR = 27.29 dB

SNR = 14 dB
PSNR = 26.63 dB

SNR = 6 dB
PSNR = 26.04 dB
Results: Performance under noise - PSF

**SNR = 40 dB**

**SNR = 20 dB**

**SNR = 14 dB**

**SNR = 6 dB**

**PSF : Xcut**

**PSF : Ycut**

**Point spread function : True Phase**

**Point spread function : Reconstructed Phase**

**Absolute difference**
Results: Performance under noise - Modulus of OTF

SNR = 40 dB

SNR = 20 dB

SNR = 14 dB

SNR = 6 dB

MTF : Xcut

MTF : Ycut

Modulus of the OTF : True Phase

Modulus of the OTF : Reconstructed Phase

Absolute difference
Residual Phase Comparison

- Principle of AO correction is to reduce the residual phase error
  \[ \Delta \phi = \phi_{turb} - \phi_{cor} \]
  - \( \phi_{cor} \): phase obtained by mirror deformation (reconstructed phase).
  - \( \phi_{turb} \): turbulent incident wavefront phase (true phase).
- Calculate \( \phi_{cor} \) using our approach and Least squares.
- Estimate \( \Delta \phi \), for the two approaches (over 1000 phase instances).
- Calculate the average PSD of \( \Delta \phi \) for the \( N = 1000 \) phase instances.
  \[ \frac{1}{N} \sum_{i=1}^{N} |\mathcal{F}(\Delta \phi)|^2 \]
- Compare the average PSD of \( \Delta \phi \) for the two approaches.
\( \phi_{\text{cor}} \) estimated with MMF using 3 different high-resolution inputs to multiresolution analysis.

Input: True Phase

- Input high-resolution phase map = True phase.

True phase

128 × 128 pixels

Comments
- Real phase to validate the performance of our approach.
- Information not available in practice.
Results

- Reconstruction over gradients of size $64 \times 64$ pixels.

No noise

SNR = 40 dB

SNR = 26 dB

SNR = 14 dB

results compared to Least squares.
Results

- Reconstruction over gradients of size $32 \times 32$ pixels.

No noise  
SNR = 40 dB  
SNR = 26 dB  
SNR = 14 dB

Superior results compared to Least squares.
Results

- Reconstruction over gradients of size $16 \times 16$ pixels.

No noise

SNR = 40 dB

SNR = 26 dB

SNR = 14 dB

- Comparable with Least squares when SNR decreases.
Input: Average Phase

- Input high-resolution phase map = Average instance of true phase.
- Averaging 10 previous and 10 post instances of the true phase.

![Average phase](image)

128 × 128 pixels

Comments

- Non-perfect high-resolution phase map as input.
Results

- Reconstruction over gradients of size $64 \times 64$ pixels.

No noise

SNR = 40 dB

SNR = 26 dB

SNR = 14 dB

Results

- Superior results compared to Least squares.
Results

- Reconstruction over gradients of size $32 \times 32$ pixels.

- No noise
- SNR = 40 dB
- SNR = 26 dB
- SNR = 14 dB

- Comparable to Least squares, superior performance under low SNR.
Results

- Reconstruction over gradients of size $16 \times 16$ pixels.

No noise

SNR = 40 dB

SNR = 26 dB

SNR = 14 dB

Results

- Comparable with Least squares when SNR decreases.
Input : FFT Phase

- Input high-resolution phase map = FFT phase-screen.
- McGlammery model for phase-screen generation using Kolmogorov PSD.

Comments

- Non-perfect high-resolution phase map as input, real scenario.
Results

- Reconstruction over gradients of size $64 \times 64$ pixels.

No noise

SNR = 40 dB

SNR = 26 dB

SNR = 14 dB

Results

Comparable to Least squares, superior performance under low SNR

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Signal Processing for Adaptive Optics
Results

- Reconstruction over gradients of size $32 \times 32$ pixels.

No noise

SNR = 40 dB

SNR = 26 dB

SNR = 14 dB

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- **PSD**: True Phase @ 128 x 128 pixels
- **Residual phase PSD**: Least squares @ 128 x 128 pixels
- **Residual phase PSD**: MMF @ 128 x 128 pixels

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Results

- Superior results compared to Least squares.
Results

- Reconstruction over gradients of size $16 \times 16$ pixels.

No noise  
SNR = 40 dB  
SNR = 26 dB  
SNR = 14 dB

Comparable with Least squares when SNR decreases.
Conclusion

- A new method for wavefront phase reconstruction in AO.
- MMF provides a suitable framework for phase estimation using multiresolution analysis.
- Superior reconstruction performance in noisy environment.
- High quality reconstruction even with non-perfect phase input.
- SE’s are ideal candidates for inferring information across scales.
  - Superior edge consistency across scales compared to classical edge detectors.
  - Better reconstruction of signal from MSM points.
Future Perspectives

- Check reconstruction with corrupted gradients. (missing pixels, measurement noise (photon noise + detector noise)).

- See the performance of reconstruction algorithm in AO system, in real-time.

- Reconstruction algorithm general to address similar problems for complex systems. (Ex: high-resolution mapping of ocean dynamics using SST maps).
Publications

Journal Publications


Peer-reviewed conferences/proceedings


Other articles scheduled.
Thank You